ON THE FLUTTER OF MULTIBAY PANELS AT LOW SUPERSONIC SPEEDS *

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Lock and Farkas[1] have investigated the flutter of a two-bay panel of infinite width in the low supersonic regime. They have determined numerically that the change in the flutter boundary from the one bay result is much smaller for clamped edges than for simply-supported edges. As Lock points out this is hardly surprising since, for the natural modes of vibration of clamped multibay plates, each bay behaves as a single bay independently of any other. It is generally appreciated that this is also true of the flutter boundary in a high supersonic flow where the aerodynamic forces are given by "piston theory". The major purpose of the present note is to demonstrate that the flutter boundaries for the multibay and one bay panels with clamped edges are identical in a low supersonic flow as well. The small difference found in Ref. 1 is attributed to the numerical approximation.

Before presenting an analytical proof of the above statement, a physical discussion is given which will make the validity of the statement plausible if not "obvious". The result is dependent primarily on the nature of supersonic flow in conjunction with the clamped edge boundary condition. Since the flow is supersonic, motion in the second bay will produce no aerodynamic pressure on the first bay. Since the first bay is also structurally decoupled from the second (clamped edges), the flutter boundary of the first bay must be the same as if there were no second bay. Now consider the second bay. Another possible motion of

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the system satisfying the condition of equilibrium and all boundary conditions is that the first bay remain motionless and the second bay behaves as a single bay. Invoking linearity, one may expect the most general motion of the system to be a superposition of those described above. Thus the flutter boundary for each bay may be expected to be the same as that of a single bay panel of the same length. The question does arise as to whether there is any other motion which is not covered by the above discussion. As will be shown by the following analysis, there is none.

ANALYSIS

The problem geometry is shown in Fig.1. The equation of motion may be written in the form

$$LW = \int_0^x K(x-\xi)W(\xi)d\xi \tag{1}$$

W is the panel deflection, L a linear differential operator and the integral arises from the aerodynamic loading. Note in particular that the deflection in the first bay, $0 < x < L_1$, induces an aerodynamic loading on the second bay, $x > L_1$. It is this fact which has led to the previous belief that the flutter boundaries for single bay and multibay panels were different even for clamped edges.

The deflection in the first bay will be denoted by W_1 and that in the second by W_2. Therefore for the first bay Eq.1 becomes

$$LW_1 = \int_0^x K(x-\xi)W_1(\xi)d\xi \tag{2}$$

and for the second

$$LW_2 = \int_{L_1}^x K(x-\xi)W_2(\xi)d\xi + \int_0^{L_1} K(x-\xi)W_1(\xi)d\xi \tag{3}$$

It will prove convenient to make transformations for the second bay as follows

$$y \equiv x - L_1$$
$$\eta \equiv \xi - L_1$$

Thus Eq.3 becomes

$$LW_2 = \int_0^y K(y-\eta) W_2(\eta)\,d\eta + \int_0^{L_1} K(y+L_1-\xi) W_1(\xi)\,d\xi \quad (4)$$

where in L, x is replaced by y. Now solve Eq.2 for W_1. We will only need to do this formally. Let

$$W_1(x) = \sum_j C_1^j W_1^j(x) \quad (5)$$

where, for definiteness, the W_1^j are the (four for a plate) independent solutions of Eq.2 satisfying the "initial" conditions

$$\frac{d^{(i-1)} W_1^j}{dx^{(i-1)}} = \delta_{ij} \qquad \begin{aligned} \delta_{ij} &= 0 \quad \text{for } i \neq j \\ &= 1 \quad \text{for } i = j \end{aligned}$$

Now solve for W_2 (again formally) from Eq.4. Let

$$W_2 = \sum_k C_2^k g^k(y) + \sum_j C_1^j f^j(y) \quad (6)$$

where $$L f^j = \int_0^y K(y-\eta) f^j(\eta)\,d\eta + \int_0^{L_1} K(y+L_1-\xi) W_1^j(\xi)\,d\xi$$

$$\text{with} \quad \frac{d^{(i-1)} f^j}{dx^{(i-1)}} = 0 \qquad \text{for all } i \text{ and } j$$

and where $$L g^k = \int_0^y K(y-\eta) g^k\,d\eta$$

$$\text{with} \quad \frac{d^{(i-1)} g^k}{dx^{(i-1)}} = \delta_{ik}$$

The first summation of Eq.6 may be considered the "homogeneous solution" and the second the "particular solution". The g^k and W_1^j are essentially the same functions.

It remains to form the eigenvalue or flutter equation by satisfying the boundary conditions on W_1 and W_2. These are:

$$W_1(0) = 0 \quad \Rightarrow \quad C_1^1 = 0 \tag{a}$$

$$\frac{dW_1(0)}{dx} = 0 \quad \Rightarrow \quad C_1^2 = 0 \tag{b}$$

$$W_2(0) = 0 \quad \Rightarrow \quad C_2^1 = 0 \tag{c}$$

$$\frac{dW_2(0)}{dy} = 0 \quad \Rightarrow \quad C_2^2 = 0 \tag{d}$$

$$W_1(L_1) = 0 \quad \Rightarrow \quad C_1^3 W_1^3(L_1) + C_1^4 W_1^4(L_1) = 0 \tag{e}$$

$$\frac{dW_1}{dx}(L_1) = 0 \quad \Rightarrow \quad C_1^3 \frac{dW_1^3}{dx}(L_1) + C_1^4 \frac{dW_1^4}{dx}(L_1) = 0 \tag{f}$$

$$W_2(L_2-L_1) = 0 \quad \Rightarrow \quad C_2^3 g^3(L_2-L_1) + C_2^4 g^4(L_2-L_1) + C_1^3 f^3(L_2-L_1) + C_1^4 f^4(L_2-L_1) = 0 \tag{g}$$

$$\frac{dW_2}{dy}(L_2-L_1) = 0 \quad \Rightarrow \quad C_2^3 \frac{dg^3}{dy}(L_2-L_1) + C_2^4 \frac{dg^4}{dy}(L_2-L_1) + C_1^3 \frac{df^3}{dy}(L_2-L_1) + C_1^4 \frac{df^4}{dy}(L_2-L_1) = 0 \tag{h}$$

Setting the determinant of coefficients to zero we have the eigenvalue equation as

$$\left[W_1^3(L_1) \frac{dW_1^4}{dx}(L_1) - W_1^4(L_1) \frac{dW_1^3}{dx}(L_1) \right]$$

$$\times \left[g^3(L_2-L_1) \frac{dg^4}{dy}(L_2-L_1) - g^4(L_2-L_1) \frac{dg^4}{dy}(L_2-L_1) \right] = 0$$

The first factor is the stability determinant for the first bay as a one bay panel and the second factor is the same for the second bay. Note the stability equation is independent of f^j. Physically the f^j are the deflections in the second bay due to the aerodynamic loading induced by the motion of the first bay. If one solves for the eigenfunction of the second bay it is found that the solution is doubly degenerate and involves two (rather than the usual one) arbitrary constants. The second bay solution is composed of a one bay solution (times an arbitrary constant) plus another solution (times an "arbitrary" constant which is determined by the first bay motion) that accounts for the deflection of the second bay under the induced aerodynamic loading of the first bay. Of course, the first bay eigenfunction is independent of the motion in the second bay.

CONCLUSIONS

The major limitations and also generalizations of the "independence property" will be stated.

First consider the limitations:

(i) It is only true for clamped edges; i.e. there can be no structural coupling between bays.

(ii) The flow must be supersonic; there can be no upstream influence.

(iii) It is only true for the flutter boundary (i.e. for linear theory); in the post-flutter (non-linear) regime the induced aerodynamic loading will change the nature of the limit cycle oscillation in the second bay from that of a single bay.

The extent to which these limitations are important can only be assessed by numerical results such as those given in Ref.1 and 2.

Finally it is apparent that the independence property holds for

(i) any number of bays, not necessarily identical

(ii) any structural element whose behavior is governed by an equation of the form of Eq.1. For example, a circular cylinder or a plate of finite width falls in this category. In the former case this arises from using a Fourier synthesis in the circumferential variable and in the latter by using a Galerkin solution in the spanwise variable.

REFERENCES

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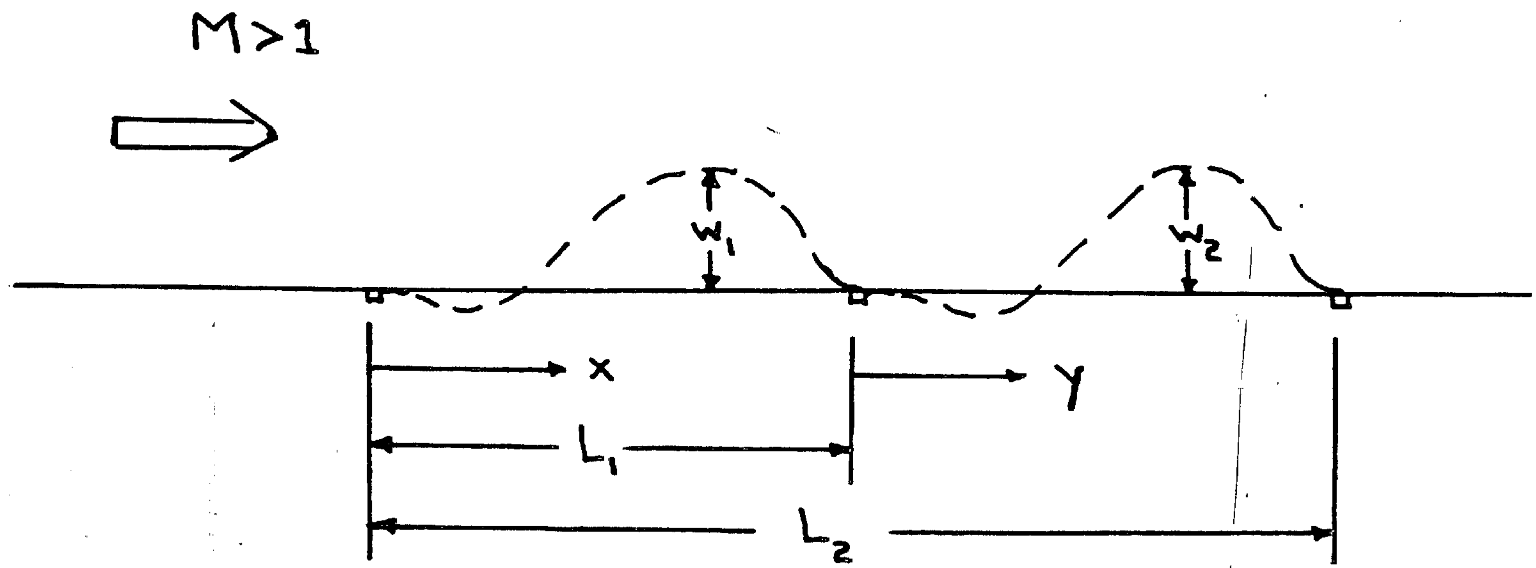

FIG. 1

Problem Geometry